***NEW POINTS ON THE GRAPH yx = xy***

Thanks to Marcelo Arruda from BRAZIL for the following ideas.

Using the basic form of De Moivre’s theorem: if then:, so if *z* is to be REAL then

must be zero so ***θ = nπ*** (ie multiples of π rads or 1800)

Therefore if is to be real then

Then, considering two **negative** real numbers “***a”*** and “***b”***, (N.B. the modulus is always positive so the **modulus** of “***a****”* is “**– *a”***) then we can write:

Now we will look at the two parts of these expressions and analyse them individually:

So, if the **positive** numbers and satisfy , then the **red** parts of above equations show this result and must be equal to each other.

(ie The equation: becomes )

The **blue** parts will be equal to each other if:

and , which means and therefore .(where is any whole number)

Recall ***a*** and ***b*** are negative so multiplying that last equality by *-1* we get (remember “***– b***” and “***– a***” are positive numbers!)

So, if we can find pairs of **positive numbers** ***x*** and ***y*** which differ by and which obey then their **opposite negative numbers**  will also satisfy the equation .

The simplest example of this is when ***x*** = 4 and ***y*** = 2. These numbers differ by 2 and they satisfy 42 = 24 so this means that the **opposites** ***x*** = – 4 and ***y*** = – 2 will also satisfy the equation: **because**

To find such numbers, we let

and solve -------------------EQU 1

for and so on.

**Examples:**

If **k = 1**, Equ. 1 becomes , whose solution is .

(Found by drawing the graphs using the AUTOGRAPH program and finding the intersection point.)

This leads to The positive solutions are **+4** and **+2**

and therefore, and will also satisfy

(In each case, the ***x*** and ***y*** values can be swapped to produce )

(We already knew these solutions.)

**Now, let’s explore some new solutions using:**

If **k = 2** (so the ***x*** and ***y*** differ by 4) then Equ 1 becomes whose solution is (from Autograph)

This leads to ,

so and will be solutions too.

Testing:

(Again we can say and are solutions)

If **k = 3** (so the ***x*** and ***y*** differ by 6) then Equ 1 becomes , whose solution is

This leads to ,

so and will be solutions too.

Testing:

(Again andare solutions too.)

If **k = 4** (so the ***x*** and ***y*** differ by 8) then Equ 1 becomes, whose solution is

This leads to ,

so and will be solutions too.

Testing:

(Again and are solutionstoo**.)**

If **k = 5** (so the ***x*** and ***y*** differ by 10) then Equ 1 becomes, whose solution is

This leads to

so and will be solutions too.

Testing:

(Again and are solutions too.)

We can continue this as far as we like, but the pattern is better seen graphically.

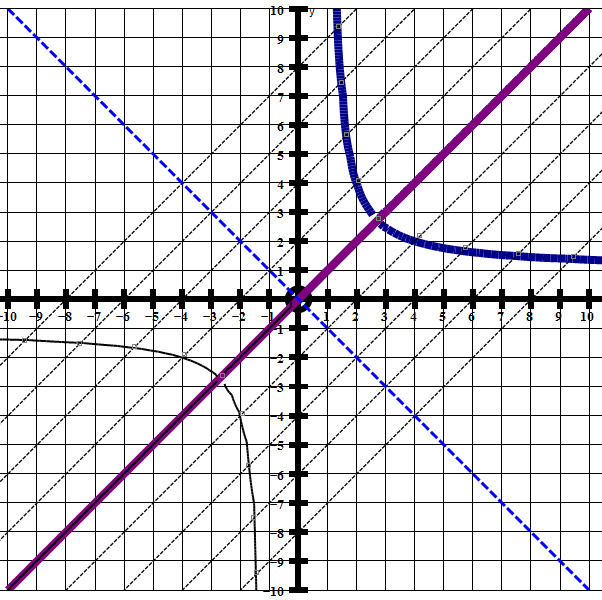
The **intersection points** ( ) of the “*hyperbola-like*” curve in the 1st quadrant, of positive ***x and y*** solutions of ***xy = yx***, with the lines ***y = x, y = x ± 2, y = x ± 4,***

***y = x ± 6*** etc., **are reflected in the line *y = – x* so that they re-appear in the 3rd quadrant but with the negative versions of the coordinates.**

We already knew the points (–4, –2), (–2.718, –2.718) and (–2, –4).

The solutions topreviously knownare **all the points on the purple line *y = x***, **all the points on the blue “hyperbola-like curve**” and all the points denoted by red dots ( )

**The LIGHT BLUE points ( ) are the new ones.**



The solutions topreviously knownare all the points on the purple line ***y = x***, all the points on the blue “hyperbola-like curve” and all the points denoted by red dots ( )